

## HUMANE MATH: Detailed Table of Contents

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## CHAPTER 1 INTRODUCTION

Are we losing our grip on reality? We act as if spending most of our time immersed in digital communication and entertainment is normal. The problem begins in elementary school, where reading, writing, and arithmetic teach the skills of symbol manipulation while ignoring the skills of living in a body and sharing in a community. *School mathematics* teaches a fantasy world of meaningless symbols, mindless computation and exact answers. Mathematics itself is similarly detached, it is built upon metaphysical principles that do not recognize common sense or communal experience.

We need to return human values to mathematics, to make math humane. *Authentic math* applies authentic tools to natural situations, it is the math that each of us uses daily and understands intuitively. *Humane math* connects to our bodies and to our common sense. We can see it working, we can feel its effects, we can experience its consequences. This type of math simply does not follow the same rules as the math that we have been taught. Our numbers do *not* go on forever!

We can begin to return human meaning to math by using mathematical forms that are *iconic rather than symbolic*. Icons look like what they mean, they connect to our bodies and to our experiences. Iconic math makes sense because it is obvious, physical and contextual. It also leads us to new ways of thinking by providing conceptual tools that emphasize unity rather than duality, ecology rather than mysticism, and the physical rather than the virtual.

## SECTION I Bringing Math Education Back to Earth

Students need to learn how to use math, not how to do math. What schools teach as math is no longer relevant to today's technological society, and for the last 50 years has not been relevant to success as a citizen in society. The resistance to teaching humane math comes from a profound confusion between math that is used and math that is idealized. The qualities of natural math include simplicity, embodiment, and sensibility.

## CHAPTER 2 AUTHENTIC MATH

## AS UNREAL AS IT GETS

Exact and Authentic Numbers

What Math Looks Like

Actual and Virtual

Math Is Not Real

Context and Meaning

## AUTHENTIC MATH EXPERIENCES

Authentic Counting

Authentic Addition

Authentic Geometry

Authentic Logic

*Authentic math* refers to the mathematical skills that people use naturally, the skills that contribute to good citizenship and to quality of life. In spite of the simplicity of authentic math, and unlike any other discipline, we begin teaching the ancient Greek theory of abstract mathematics in early grade school. Authentic numbers are estimates, they are rarely exact. Authentic counting rarely goes past 100. Authentic addition works differently in different contexts. Authentic geometry is anchored to physical measurement. Authentic logic does not require symbolic deduction.

## CHAPTER 3 SCHOOL MATH

MATH ANXIETY

MATH TESTING

Impossible Measurements

Testing Is Irrelevant

THREE FACES OF SCHOOL MATH

How School Is Confused

A Math Education Rant

An Elegant Solution

Math neurosis is caused by forcing students to learn what they know intuitively to be incorrect. Math testing is a primary source of math anxiety. Our obsession with educational testing and measurement violates psychological, educational and mathematical principles. Math is an academic discipline, a pragmatic tool and an educational activity. Math education can be improved and math anxiety eliminated by teaching math as a useful tool rather than as an abstract discipline. Math comes in pure, applied and computational versions. We teach Pure Math in first grade by chanting numbers and memorizing tables. This is immediately followed by Computational Math, memorizing counter-intuitive algorithms of addition and multiplication. To be both effective and humane, schools should teach Applied Math, the math that was used naturally and intuitively prior to the twentieth century, before symbolic math cut us off from our cultural and evolutionary roots. Pure and Computational Math should be reserved for students who are math and computer science majors in college. The more that symbolic math becomes dominant in the classroom, the more that students and teachers become estranged from the psychology, physiology and sociology of learning.

## CHAPTER 4 WHAT MAKES SENSE

WHAT IS A FRACTION?

HOW PEOPLE LEARN

Simplicity

Embodiment

Sensibility

Uniqueness

CURRICULUM RECAPITULATION

PLURALISM AND CONTEXT

Fractions exemplify the non-sense math that, outside of school, our society protects us against. The kind of math that people value is simple to use and to understand, it is intuitive and easily

learned, and it has a visceral connection to the body. This math makes common sense because it has co-evolved with the growth of commerce and civilization. It arises from authentic problems. It is interpreted in a relevant context. It is grounded in our bodies, our activities, and our world.

## CHAPTER 5 ETHNOMATHEMATICS

POLITICAL PLAYTHINGS  
 INDOCTRINATION  
 CULTURAL IMPERIALISM  
 SOCIAL MEDIA

Mathematics is a cultural artifact. Authentic math is used extensively by indigenous cultures. Our current classrooms use European mathematical abstraction as a form of cultural indoctrination. Math as it is taught is based upon a dualistic worldview, a quasi-theological mysticism, and a failure to understand the unity of ideas and actions within a physical environment. Not only have schools stripped math education of its human associations, they have also isolated math learning from the societal contexts that produce meaning. Almost all pre-college math textbooks contain seriously misleading information about what math is and how math actually works. The vigorous debate over math education addresses only secondary issues. At the root of the problem is the structure of mathematics itself.

## CHAPTER 6 THE FUTURE WAS YESTERDAY

TECHNOLOGICAL INTEGRATION  
 MY CALCULATOR, MY BODY  
 EXPONENTIAL CHANGE  
 WICKED PROBLEMS

Computer technology has permeated every facet of our society, and it has redefined what people need to know about math. Manipulation of symbols is reliably provided by machines. Multiplying numbers by hand is as anachronistic as a land-line rotary telephone, memorizing math algorithms as irrelevant as horse-drawn carriages. Digital convergence puts sophisticated mathematical tools into everyone's hands. The delegation of computation to computers can help classroom math to become authentic. For the problems currently facing society, math can provide information but not solutions.

## SECTION II Seeking Humane Mathematics

We move from math education to math itself, examining what math is, how it works, and what it requires us to believe. Why is math afforded the privileged status of being embodied within Nature? The origins of simple arithmetic illustrate how our current use of mathematics has abandoned its physiological foundations. The ancient Greeks embraced the metaphysical trinity of Zero, One, Infinity, while only One in this trinity actually exists. A century ago, *symbolic formalization* removed human intuition from human "figuring", while at the same time validating the values of the Platonic world of perfect abstraction: *the concepts of math do*

*not require time, space, matter, resources or reality.* The primary message of math education is that a student should not recognize our shared physical world. The Section includes a quick look at some modern maths that are beginning to return to this world. Silicon computation in particular has eroded the veneer of abstraction that protects mathematics from reality.

## CHAPTER 7 NUMBERS

HOW COUNTING BEGAN

WHAT ARE NUMBERS?

What Is One? What Are Two?

Roman-Hindu-Arabic Numerals

Types of Numbers

Utility and Quality

Design Criteria for Numerals

WHAT IS ARITHMETIC?

SCIENTIFIC NOTATION

Our notation for numbers has evolved while the meaning of numbers and arithmetic has devolved, from natural human activities to meaningless symbol manipulation. The meaning of numbers and arithmetic can be retrieved from historical records. The design of number systems includes a tradeoff between human utility (the human user in a physical context) and machine utility (context-free mechanical manipulation). Some numerical concepts are essential, some are artifacts of how we write numbers down, and some are simply props to support abstract numerical mysticism.

## CHAPTER 8 HOW MATH WORKS

WHAT IS MATHEMATICS?

WHAT WAS MATHEMATICS?

NUMERIC AND SYMBOLIC COMPUTATION

MATHEMATICA

PERSPECTIVES

Utility

Computation

Neuroscience

Language

Social Meme

Art

Trauma

Until recently, mathematical ideas and applications co-evolved, especially in the physical sciences. Is math “out there”? Is it part of the human psyche? Why do people believe that math works? We explore different ways to think about math as a cultural phenomenon. Math is an enlightened exploration of pure abstraction, a powerful tool, a foundation for computing, a way of thinking, a way of talking, a method used to control people and events, a thing of beauty, and a rocky road of blinding fantasies and broken beliefs.

## CHAPTER 9 PHILOSOPHIES OF MATHEMATICS

MATHEMATICAL KNOWLEDGE  
 PLATONISM AND CONSTRUCTIVISM  
 SYMBOLIC ABSTRACTION  
   The Cartesian Split  
   Hilbert's Program  
   Canons of Formal Systems  
   Using a Formal System  
 THE MAGIC OF MATH  
 UNCERTAINTY

The philosophical connections between humanity, math, and reality are thousands of years old. They have been rendered irrelevant by automated computation. Historically, religion and math are deeply intertwined. Mathematics continues to embrace mystical concepts, presenting them to an unsuspecting public as “rational thinking”. We trace the evolution of math as a collective fantasy of perfection that often has proved to be excessively fragile. The concept of formalization is founded in Cartesian dualism (the mind is separate from the body). Formal systems legitimize the virtuality of symbolic abstraction as a direct competitor to experience within physical reality.

## CHAPTER 10 COMPUTATION

TOWER OF BABEL  
 RECURSION AND CONSTRAINT  
 PROGRAMMING  
 INTERFACE  
 REALITY AS COMPUTATION  
   Circuitry  
   Adders  
   Artificial Intelligence  
   Computational Physics

Although computers currently dominate our interaction with reality, the mechanism used in all computers is very simple: *timed elementary logic*. The plasticity of electronic media lets us confound this simple form of math with authentic experience. The ubiquity of computational devices is blinding us to the fact that the earth is not an equation, that the universe is not a computer, that perception is not a pixel display, and that humanity cannot survive without attention to the physical. Three computational disciplines illustrate the illusions of virtuality. Silicon circuitry shares nothing in common with biological systems. Artificial intelligence simply debases the concept of human intelligence. Computational physics fails the humility test.

## CHAPTER 11 DIAGRAMMATIC MATH

MODERN ALGEBRA  
 DIAGRAMMATIC SYSTEMS  
   Visual Objects

Parallelism  
 ICONIC MATH  
 Cellular Automata  
 Fractals  
 Tally Arithmetic  
 Venn Diagrams and Existential Graphs

A quick tour through some new mathematical ideas illustrates the recent evolution of non-symbolic math. Mathematical concepts can be expressed as easily in spatial patterns as in symbolic patterns. The limits of computing, the resurgence of geometry, the facilitation of parallelism, and the visual and interactive nature of computer graphics have each redefined what math might be. Cellular automata explore the possible social networks between symbols. Fractals demonstrate that complexity arises from lots of simplicity. Diagrammatic counting, arithmetic, algebra, logic and set theory each abandons the string notation of symbols to show that the essential concepts of mathematics are much simpler than we have been taught.

### SECTION III Getting Lost in Virtuality

We humans abstract and simplify, in so doing we construct virtual worlds. The symbolic arts of language and mathematics have led the way to a most profound alienation, an abandonment of reality itself, first through *symbolic detachment*, then through *virtualization*. We believe that dwelling within symbols and images will save us from our own bodies. Both math and media technologies seduce us into thinking that virtuality is as important as reality. Both math and media ignore the body while encouraging fantasies of the mind. At the core of the virtuality constructed by modern media is *mathematical-mediation*, the conversion of authentic experience into digital electronic experience via patterns of binary symbols. The intimacy between math and media is exemplified by virtual reality technologies.

### CHAPTER 12 SYMBOL/ICON

SYMBOLISM  
 SYMBOLIC AND ICONIC FORMS  
 MEANING AND REPRESENTATION  
 INFORMATION PROCESSING  
 INFORMATION MAPPING  
 CONSCIOUSNESS

The currency of mathematics, like the currency of literature, is strings of symbols. In the twentieth century, those who believed in diagrams and meaningful visual information battled those who believed in symbols and meaningless symbolic strings. Meaning was victorious in the form of the written word, meaning was defeated by the form of math. Symbolic patterns limit understanding; the most limiting perspective is to project the behavior of silicon machines onto human thinking and consciousness.

## CHAPTER 13 ELECTRONIC MEDIA

MEDIA  
 MEDIATION  
 DIMENSION  
 INCLUSION  
 VIRTUAL SEMANTICS

Our waking hours are dominated by the electronically mediated virtuality of media. Media condense physical reality by projecting it into lower dimensions, by ignoring the substance of experience. Media reduce what it is to be human to a caricature, to an abstraction. The discipline of mathematics established the model for this self-denigrating behavior by separating meaning from experience. The resulting *virtual semantics* creates a now dominant illusion that interaction with electronic media is interaction with reality. In the case of math, the illusion is that an extremely condensed symbolic structure can simulate authentic experience.

## CHAPTER 14 INSIDE INFORMATION

VIRTUAL REALITY  
 THE VIRTUAL BODY  
 IMMERSION  
 Interaction  
 Designing  
 Teaching  
 Thinking  
 Artificial Life  
 Experiential Math  
 THE MATHEMATICS OF VIRTUALITY

The thoughts and ideas that drove the initial development of virtual reality systems in the late 1980s provide a recent extreme example of symbolic detachment. Virtual worlds abandon the constraints of the body, making psychology the physics of VR. We long to have the capabilities of superheroes, we are delighted to be able to escape the confines of physical reality. Underneath virtuality is mathematics; ultimately it is mathematical abstraction that allows us to ignore the responsibilities of the body and the earth.

## CHAPTER 15 VIRTUALITY

THE AGE OF VIRTUALITY  
 SIMPLE SPACE  
 TAKING NOTHING SERIOUSLY  
 THE FOUNDATIONS OF BOUNDARY MATH  
 Void  
 Distinction  
 Participation  
 Marking  
 The Forbidden  
 SELF-REFERENCE  
 SUMMARY OF TOOLS



When we make a distinction, we create *context* for interpretation and *content* for abstraction. We are direct participants in the construction of both reality and virtuality, and we are beginning to understand how to completely immerse ourselves within the virtual. Boundary math provides a formal language and concept system that describes the human interface with virtuality. A central concept is the void, nothing at all. Self-reference provides a bridge between authentic and virtual experience.

## SECTION IV Building a Humane Foundation

Math education reinforces our cultural belief that math must be symbolic and incomprehensible to be rigorous. *Boundary math* shows that math can be both formal and humane. Unit ensembles remind us that arithmetic is essentially simple, it is our symbolic notation that is confusing. Depth-value shows that arithmetic does not require symbol manipulation to be efficient. Spatial algebra renders most of the rules of symbolic algebra irrelevant. Iconic logic shows how symbolic logic is not only clumsy and misleading, it is also a poor model for rational thinking. Boundary thinking is a new form of *humane rationality* based on the conceptual structure of iconic math. Its void-based eco-rational foundations reject Cartesian duality, mathematical mysticism, and the presumption that resources of any kind are unlimited.

### CHAPTER 16 BOUNDARY MATH

#### A NEW KIND OF ELEMENTARY MATH

Math from the Beginning

Starting Simply

The Page Icon

We Are Now Leaving Reality

Discovering Complexity

Representation and Replication

Constructing a Language

To Be Equal

Laws of Form

#### CONSTRUCTING BOUNDARY MATH

Boundary Arithmetic

Counting

Adding

Grouping

Multiplying

Simple Rules

Boundary Logic

Arithmetic and Logic Together

Recapitulation

Boundary math is iconic rather than symbolic. It appeals to our senses rather than to our capacity to memorize. It incorporates the perspective of its user, the contextual relevance of information, and the physicality of computation without abandoning the rigor that characterizes mathematical systems. Boundary arithmetic and boundary logic are described with an emphasis on the experiential nature of these forms. Unit ensembles are tallies, historically the first notation for numbers. Depth-value notation makes tallies easy to read and to compute with. Iconic logic provides a more direct route to rational thought. Boundary math demonstrates that common mathematics arises naturally from the primitive act of communication. A consequence is that math is not an embodiment of reality.

## CHAPTER 17 UNIT-ENSEMBLES

ADDITIVE SYSTEMS

ICONIC NOTATION

CALCULATING WITHOUT COUNTING

MEREOLGY

ASSUMPTIONS AND MECHANISMS

Adding and Subtracting

Multiplying

Reciprocals, Fractions, and Dividing

EXAMPLES OF BOUNDARY ARITHMETIC

Unit ensembles provide an extended example of the evolutionary design of a number system for human consumption. Unit ensemble arithmetic returns the Additive Principle (a sum is the collection of its parts) to the act of addition. By adopting the principles of tally arithmetic in use since the dawn of civilization, counting and adding and multiplying become intuitive and effortless. Unit ensembles are built upon activity rather than concept. Several examples of boundary arithmetic systems are described.

## CHAPTER 18 DEPTH-VALUE

POSITIONAL NOTATION

BOUNDARY NUMBERS

FACTORING

PARALLEL ARITHMETIC

Place-value notation was created to make arithmetic feasible for large ensembles. It is also clumsy and hard to learn. Positional notation forces a sequential approach to computation, making simple addition and multiplication unnecessarily complicated. Depth-value notation maintains the advantages of tally arithmetic: it is visual, parallel, intuitive, and simple. Unit ensembles expressed in depth-value notation reunite intuition with numerical computation.

## CHAPTER 19 ICONIC ALGEBRA

ALGEBRAIC ABSTRACTION

ICONIC FOUNDATIONS

EXAMPLES OF ICONIC ALGEBRA

Spatial Algebra

Boundary Algebra  
 Exponents  
 Differentiation  
 RE-ENTERING YOUR OWN SPACE  
 Contradiction  
 Imaginaries  
 A New Imaginary Number

Algebra shows us the symmetry and structure of mathematical objects. The direct connection of this structure to authentic experience permits algebra to unite with physical activity. Several iconic forms of algebra are described. When time and self-reference are incorporated within mathematical computation, contradiction and inconsistency arise naturally. Boundary notation provides new insight into the mathematics of imaginary values.

## CHAPTER 20 ICONIC LOGIC

CLASSICAL LOGIC  
 ICONIC FOUNDATIONS  
 BOUNDARY LOGIC  
 Crossing and Calling  
 Dominion, Involution, Pervasion  
 Void-equivalence and Transparency  
 Deduction as Containment  
 NEW PERSPECTIVES  
 UNARY PHILOSOPHY  
 BOUNDARY THINKING

Over millennia logic has accumulated detritus that a clearer point of view can eliminate. We don't need the dualism of True/False, the juggling and rearrangement of meaningless symbols, the accumulation of facts, the separation of behavior from cognition, or the decontextualization of logic in order to think clearly. Boundary logic removes both the symbolic redundancy and the linear perspective from classical logic, leading to a logic without dualism and without the arduous manipulations of symbolic proof. Clear thinking rests upon forgetting what is irrelevant. Deduction rests upon deletion, not accumulation, of information.

## CHAPTER 21 SUMMING IT UP

We have explored how math has encouraged us to abandon reality, and how our educational institutions are confusing utility with fantasy. We've identified how math is built upon metaphysical foundations at the cost of common sense, and how math is leading us not to reality but to virtuality. Mathematics needs to return to earth in order to help the Earth. Of course, I do not expect modern math to change. Rather, the purpose is to provide a rigorous alternative that reunites human comprehension with number sense and with visceral intuition. An iconic mathematics grounded in physical activity and common experience can not only reduce math neurosis, it can provide new conceptual tools that support a sane future.